ges) **Reg. No. :** ....

de No. : 30566 E **Sub. Code : AMMA 21 / CMMA 21**

c. (CBCS) DEGREE EXAMINATION, APRIL 2022.

Second Semester

Mathematics — Core

FFERENTIAL EQUATIONS AND ANALYTICAL GEOMETRY OF THREE DIMENSIONS

(For those who joined in July 2020 onwards)

e : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

The complete solution of $y = px + p^2$ where $\left(p = \frac{dy}{dx}\right)$ is ————.

(a) $y = x^2 + c$ (b) $y = cx^2 - c$

(c) $y = cx + c^2$ (d) $cx - c$

2. The general solution of $(D^2 - 4)y = 0$ is $y =$ ————.

(a) $Ae^{2x} + Be^{-2x}$ (b) $Ae^{4x} + Be^{-4x}$

(c) $Ae^{3x} + Be^{x}$ (d) $Ae^{4x} + B$

3. The particular integral of $(D^2 - 9)y = \cos 3x$ is ————

(a) $\frac{\cos 3x}{18}$ (b) $\frac{\cos 3x}{9}$

(c) $\frac{\cos 3x}{-18}$ (d) 0

4. The solution of the differential equation $p^2 - 9p + 18 = 0$ where $p = \frac{dy}{dx}$ is ————

(a) $(y - 3x - c)(y - 6x - c) = 0$

(b) $(y - 6y - c)(y - 3x - c) = 0$

(c) $(x - 6y - c)(3x - y - c) = 0$

(d) $x^2 - 9x + 18 = 0$

5. The direction ratios of the line joining (1,2,–1) and (2, –1, 1) are ————.

(a) 2, 6, 4 (b) 1, –3, 2

(c) –2, –6, –4 (d) –1, 3, –2

The equation of the plane through (1,0,2) and parallel to the plane $2x + 3y - 4z = 0$ is ————

(a) $3x + 2y - 3z + 6 = 0$ (b) $3x + 2y - 3z - 6 = 0$

(c) $2x + 3y - 4z + 6 = 0$ (d) $2x + 3y - 4z - 6 = 0$

The line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z-4}{-1}$ is parallel to the plane ————

(a) $x - 2y - 4z + 7 = 0$ (b) $2x - 2y - 4z + 7 = 0$

(c) $x - 7y - 4z + 7 = 0$ (d) $7x - 7y - 4z + 7 = 0$

If the line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ ———— the plane $ax + by + cz + d = 0$ then $al + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d = 0$.

(a) lies in (b) is parallel to

(c) bisects (d) is proportional to

Centre and radius of the sphere $x^2 + y^2 + z^2 - 6x - 2y - 4z - 11 = 0$ is

(a) (0,2,4) and 16 (b) (0,–1,2) and –4

(c) (3,1,2) and 5 (d) (1,–1,2) and –6

The equation of the tangent plane at the origin to the sphere $x^2+y^2+z^2-8x-6x+4z=0$ is ————

(a) $4x + 3y - 2z = 0$ (b) $4x - 3y - 2z = 0$

(c) $4x - 3y + 2z = 0$ (d) $-4x + 3y + 2z = 0$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve: $x^2(y - px) = yp^2$.

Or

(b) Solve: $p^2 + 2py\cot x - y^2 = 0$.

12. (a) Solve: $(D^3 - 3D^2 + 3D - 1)y = x^2 e^x$.

Or

(b) Solve: $x^2 y'' + 3xy' + y = \frac{1}{(1-x)^2}$.

13. (a) The line joining A(4,3,2) and B(1,2,–3) meets the planes YOZ, XOY in C,D respectively. Find the coordinates of C and and D and the ratios in which they divide AB.

Or

(b) Find the equation of the plane through the line of intersection of the plane $2x + y + 3z - 4 = 0$ and $4x - y + 5z - 7 = 0$ and perpendicular to the plane $x + 3y - 4z + 6 = 0$.

14. (a) Find the perpendicular distance form $(3,9,-1)$ to the line $\frac{x+8}{-8}=\frac{y-31}{1}=\frac{z-13}{5}$

Or

(b) Find the equations of the plane passing through the line $5x-2y+7=0=x-3y+z-4$ and parallel to the line $\frac{x}{2}=\frac{y}{1}=\frac{z-1}{-2}$.

15. (a) Find the equation to the sphere through the four points $(0,1,3)$, $(1,2,4)$, $(2,3,1)$ and $(3,0,2)$

Or

(b) Find the equation of the tangent line to the circle $x^2+y^2+z^2-x+4z=0$; $3x-2y+4z+1=0$ at the point $(1,-2,-2)$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve: $\frac{dx}{dt}-\frac{dy}{dt}+x-y=1$

$$2\frac{dx}{dt}+\frac{dy}{dt}=t$$

Or

(b) Solve: $(px-y)(x+yp)=a^2p\ (x^2=x,\ y^2=y)$.

17. (a) Solve: $(D^2+1)y=x^2e^2x+x\cos x$.

Or

(b) Apply the method of variation of parameters to solve $y''=3y'=2y=x^2$

18. (a) A line makes an angles $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$.

Or

(b) Show that the origin lies in the acute angle between the planes $x+2y+2z=9$, $4x-3y+12z+13=0$. Find the planes bisecting the angles between them and point out which bisects the obtuse angle.

19. (a) Find the equations of the image of the line $\frac{x-1}{2}=\frac{y+2}{-5}=\frac{z-3}{2}$ in the plane $2x-3y+2z+3=0$.

Or

(b) Show that the lines $\frac{x-5}{4}=\frac{y-7}{4}=\frac{z+3}{-5}$ and $\frac{x-8}{7}=\frac{y-4}{1}=\frac{z-5}{3}$ are coplanar. Find their common point and find the equation of the plane which they lie.

20. (a) Find the equation of the sphere which passes through the circle $x^2+y^2+z^2-2x-4y=0$ $x+2y+3z=8$ and touches the plane $4x+3y=25$

Or

(b) Show that the conditions for the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ to cut the sphere $x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ in a great circle is $2uu_1+2vv_1+2ww_1-(d+d_1)=2r_1^2$ where r_1 is the radius of the latter sphere.